

Complexity Results for Manipulation, Bribery and Control of the Kemeny Procedure in Judgment Aggregation

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Abstract

We study the computational complexity of several scenarios of strategic behavior for the Kemeny procedure in the setting of judgment aggregation. In particular, we investigate (1) manipulation, where an individual aims to achieve a better group outcome by reporting an insincere individual opinion, (2) bribery, where an external agent aims to achieve an outcome with certain properties by bribing a number of individuals, and (3) control (by adding or deleting issues), where an external agent aims to achieve an outcome with certain properties by influencing the set of issues in the judgment aggregation situation. We show that determining whether these types of strategic behavior are possible (and if so, computing a policy for successful strategic behavior) is complete for the second level of the Polynomial Hierarchy. That is, we show that these problems are Σ_2^P -complete.

1 Introduction

An important topic in the research field of computational social choice is the (im)possibility of strategic behavior in collective decision making. This is epitomized by the eminence of results such as the Gibbard-Satterthwaite Theorem [16, 22], that identifies various conditions under which strategic voting (or manipulation) is, in principle, unavoidable. Manipulation in voting is a typical example of strategic behavior, and involves individuals reporting insincere preferences with the aim of obtaining a group outcome that is preferable for them.

Since strategic behavior in collective decision making is generally considered to be (socially) undesirable, a lot of research effort has been invested in diagnosing what social choice procedures are resistant to strategic behavior, and under what conditions. An important research direction along these lines investigates how computational complexity can be used to establish that various social choice procedures are (in many cases) practically immune to strategic behavior [2, 5]. For example, in many cases, it is in principle possible to manipulate voting rules (by reporting insincere preferences), but determining what insincere preference leads to a better outcome is computationally so demanding that it prevents manipulative behavior from being a useful policy.

Contributions In this paper, we use the framework of computational complexity theory to study several scenarios of strategic behavior in the setting of judgment aggregation. Judgment aggregation studies collective decision making on a set of issues that are logically related [12]. In particular, we study three scenarios of strategic behavior for the *Kemeny judgment aggregation procedure*—which is one of the most prominent judgment aggregation procedures known from the literature. We investigate:

1. *manipulation*, where an individual reports an insincere individual judgment in an attempt to enforce a preferable group judgment (from their point of view);

2. *bribery*, where an external party bribes several individuals that are involved in the group decision process (that is, the briber stipulates their individual judgments) in order to obtain a group judgment with certain properties; and
3. *control*, where an external party controls the set of issues that are involved in the judgment aggregation setting, with the aim of achieving a group judgment with certain properties.

Concretely, we study various different decision problems that formalize the computational tasks involved in the strategic behavior in each of these scenarios. We show that all the computational problems that we consider in this paper are Σ_2^P -complete. That is, we show that:

- Manipulation for the Kemeny rule in judgment aggregation is Σ_2^P -complete.
- Bribery for the Kemeny rule in judgment aggregation is Σ_2^P -complete.
- Control (by adding or removing issues) for the Kemeny rule in judgment aggregation is Σ_2^P -complete.

(Completeness for the complexity class Σ_2^P indicates that a problem is computationally intractable. Even the easier problem of checking whether a given candidate solution is in fact a solution is not efficiently solvable—it requires solving an NP-complete problem.)

Various different frameworks have been used in the literature to formalize the setting of judgment aggregation (see, e.g., [13]). The computational complexity results that we develop hold for two commonly considered judgment aggregation frameworks: *formula-based judgment aggregation* and *constraint-based judgment aggregation*. We discuss these judgment aggregation frameworks in more detail in Section 2.2. (In order to capture the scenario of control naturally in the constraint-based judgment aggregation framework, we consider a slightly extended variant of this framework. For more details, see Section 2.2.3.)

Most of the various forms of strategic behavior that we consider in this paper involve the incentive of achieving a *preferable* group outcome. There are various ways to define preference relations over (individual and group) judgments. The preferences that we study are based on weighted Hamming distances. That is, we consider weight functions that assign to each issue a weight that indicates how important it is for an individual (or for an external party) that the group judgment agree with their judgment on this issue. Such weight functions naturally induce preference relations over judgments.

In addition, we study variants of the strategic behavior scenarios where the objective is to obtain a group judgment that includes a given set of conclusions. This can be seen as an all-or-nothing variants (the group outcome either includes the required set of conclusions or it does not), whereas the variants involving preferences based on weighted Hamming distances offer a more gradual view (maybe the optimal outcome is not possible, but the current outcome can still be improved slightly by behaving strategically).

Worst-case Complexity The computational intractability results that we provide in this paper can be seen as positive results, since they show that various kinds of undesirable strategic behavior cannot be used efficiently across the board due to computational complexity obstructions. However, it is important to emphasize that the computational complexity results that we provide in this paper are *worst-case complexity* results. Worst-case intractability results indicate that there is no algorithm that works efficiently in all possible cases. However, it might well be the case that there are restricted settings where several forms of strategic behavior are efficiently possible.

In order to consolidate the conclusion that strategic behavior for the Kemeny procedure in judgment aggregation is computationally intractable, further research is needed. Such further research would have to establish that the various forms of strategic behavior remain computationally intractable in many restricted settings. A key tool for establishing computational complexity results for restricted settings is the paradigm of parameterized complexity [10, 11, 15, 20]—this is a framework where the complexity of computational problems is measured in a multi-dimensional way, in contrast to the classical theory of computational complexity, where the complexity of problems is measured only in terms of the input size in bits.

Related Work The concept of manipulation in judgment aggregation has been studied before in the literature, both from an axiomatic point of view [4, 6, 8] and from a computational complexity point of view [3, 14]. The complexity analysis of manipulation in judgment aggregation that has been done in the literature is restricted to uniform premise-based quota rules. Additionally, bribery in judgment aggregation has been studied from a computational complexity point of view for uniform premise-based quota rules [3].

Outline We begin in Section 2 by considering relevant notions from computational complexity and judgment aggregation that we use in this paper. Then, in Section 3, we develop the intractability results for the scenario of manipulation. In Section 4, we turn to the scenario of bribery, and in Section 5, we consider the scenario of control (by adding or deleting issues). Finally, we conclude in Section 6.

2 Preliminaries

Before we turn to the complexity results that we develop in this paper, we review several relevant concepts from computational complexity theory and judgment aggregation.

2.1 Complexity Theory

We begin with reviewing some basic notions from computational complexity. We assume the reader to be familiar with the complexity classes P and NP, and with basic notions such as polynomial-time reductions. For more details, we refer to textbooks on computational complexity theory (see, e.g., [1]).

We briefly review the classes of the Polynomial Hierarchy (PH) [19, 21, 23, 24]. In order to do so, we consider quantified Boolean formulas. A *(fully) quantified Boolean formula (in prenex form)* is a formula of the form $Q_1x_1Q_2x_2\ldots Q_nx_n.\psi$, where all x_i are propositional variables, each Q_i is either an existential or a universal quantifier, and ψ is a (quantifier-free) propositional formula over the variables x_1, \ldots, x_n . Truth for such formulas is defined in the usual way.

To consider the complexity classes of the PH, we restrict the number of quantifier alternations occurring in quantified Boolean formulas, i.e., the number of times where $Q_i \neq Q_{i+1}$. We consider the complexity classes Σ_k^P , for each $k \geq 1$. Let $k \geq 1$ be an arbitrary, fixed constant. The complexity class Σ_k^P consists of all decision problems for which there exists a polynomial-time reduction to the problem QSAT_k, that is defined as follows. Instances of the problem QSAT_k are quantified Boolean formulas of the form $\exists x_1 \ldots \exists x_{\ell_1} \forall x_{\ell_1+1} \ldots \forall x_{\ell_2} \ldots Q_k x_{\ell_{k-1}+1} \ldots Q_k x_{\ell_k} . \psi$, where $Q_k = \exists$ if k is odd and $Q_k = \forall$ if k is even, where $1 \leq \ell_1 \leq \ldots \leq \ell_k$, and where ψ is quantifier-free. The problem is to decide if the quantified Boolean formula is true. The complementary class Π_k^P consists of all decision problems for which there exists a polynomial-time reduction to the problem co-QSAT_k, that is complementary

to the problem QSAT_k . The Polynomial Hierarchy (PH) consists of the classes Σ_k^P and Π_k^P , for all $k \geq 1$.

Alternatively, one can characterize the class Σ_2^P using nondeterministic polynomial-time algorithms with access to an oracle for an NP-complete problem. Let O be a decision problem. A Turing machine \mathbb{M} with access to an O oracle is a Turing machine with a dedicated *oracle tape* and dedicated states q_{query} , q_{yes} and q_{no} . Whenever \mathbb{M} is in the state q_{query} , it does not proceed according to the transition relation, but instead it transitions into the state q_{yes} if the oracle tape contains a string x that is a yes-instance for the problem O , i.e., if $x \in O$, and it transitions into the state q_{no} if $x \notin O$. Intuitively, the oracle solves arbitrary instances of O in a single time step. The class Σ_2^P consists of all decision problems that can be solved in polynomial time by a nondeterministic Turing machine that has access to an O -oracle, for some $O \in \text{NP}$.

2.2 Judgment Aggregation

Next, we introduce the two formal judgment aggregation frameworks that we use in this paper: *formula-based judgment aggregation* (as used by, e.g., [7, 14, 18]) and *constraint-based judgment aggregation* (as used by, e.g., [17]). Moreover, we briefly discuss an extended variant of the constraint-based judgment aggregation framework (as considered in, e.g., [13]).

2.2.1 Formula-Based Judgment Aggregation

We begin with the framework of formula-based judgment aggregation.

An *agenda* is a finite, nonempty set Φ of formulas that does not contain any doubly-negated formulas and that is closed under complementation. Moreover, if $\Phi = \{\varphi_1, \dots, \varphi_n, \neg\varphi_1, \dots, \neg\varphi_n\}$ is an agenda, then we let $[\Phi] = \{\varphi_1, \dots, \varphi_n\}$ denote the *pre-agenda* associated to the agenda Φ . We denote the bitsize of the agenda Φ by $\text{size}(\Phi) = \sum_{\varphi \in \Phi} |\varphi|$. A *judgment set* J for an agenda Φ is a subset $J \subseteq \Phi$. We call a judgment set J *complete* if $\varphi \in J$ or $\neg\varphi \in J$ for all $\varphi \in \Phi$; and we call it *consistent* if there exists an assignment that makes all formulas in J true. Intuitively, the consistent and complete judgment sets are the opinions that individuals and the group can have.

We associate with each agenda Φ an integrity constraint Γ , that can be used to further restrict the set of feasible opinions. Such an *integrity constraint* consists of a single propositional formula. We say that a judgment set J is Γ -consistent if there exists a truth assignment that simultaneously makes all formulas in J and Γ true. Let $\mathcal{J}(\Phi, \Gamma)$ denote the set of all complete and Γ -consistent subsets of Φ . We say that finite sequences $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ of complete and Γ -consistent judgment sets are *profiles*, and where convenient we equate a profile $\mathbf{J} = (J_1, \dots, J_p)$ with the (multi)set $\{J_1, \dots, J_p\}$. Moreover, for $i \in [p]$, we let \mathbf{J}_{-i} denote the profile $(J_1, \dots, J_{i-1}, J_{i+1}, \dots, J_p)$.

A *judgment aggregation procedure* (or *rule*) for the agenda Φ and the integrity constraint Γ is a function F that takes as input a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, and that produces a non-empty set of non-empty judgment sets. We call a judgment aggregation procedure F *resolute* if for any profile \mathbf{J} it returns a singleton, i.e., $|F(\mathbf{J})| = 1$; otherwise, we call F *irresolute*. We call a judgment aggregation procedure F *anonymous* if for every profile $\mathbf{J} = (J_1, \dots, J_p)$ and for every permutation $\pi : [p] \rightarrow [p]$ it holds that $F(\mathbf{J}) = F(\mathbf{J}')$, where $\mathbf{J}' = (J_{\pi(1)}, \dots, J_{\pi(p)})$. An example of a resolute, anonymous judgment aggregation procedure is the *strict majority rule* Majority, where $\text{Majority}(\mathbf{J}) = \{J^*\}$, where $\varphi \in J^*$ if and only if φ occurs in the strict majority of judgment sets in \mathbf{J} , for all $\varphi \in [\Phi]$, and where $\varphi \in J^*$ if and only if $\neg\varphi \notin J^*$, for all $\varphi \in \Phi$. We call a judgment aggregation procedure F *complete* and Γ -consistent, if J is complete and Γ -consistent, respectively, for every $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ and every $J \in F(\mathbf{J})$. The procedure Majority is not consis-

tent. Consider the agenda Φ with $[\Phi] = \{p, q, p \rightarrow q\}$, and the profile $\mathbf{J} = (J_1, J_2, J_3)$, where $J_1 = \{p, q, (p \rightarrow q)\}$, $J_2 = \{p, \neg q, \neg(p \rightarrow q)\}$, and $J_3 = \{\neg p, \neg q, (p \rightarrow q)\}$. The unique outcome $\{p, \neg q, (p \rightarrow q)\}$ in $\text{Majority}(\mathbf{J})$ is inconsistent.

The *Kemeny aggregation procedure* is based on a notion of distance. This distance is based on the Hamming distance $d(J, J') = |\{\varphi \in [\Phi] : \varphi \in (J \setminus J') \cup (J' \setminus J)\}|$ between two complete judgment sets J, J' . Intuitively, the Hamming distance $d(J, J')$ counts the number of issues on which two judgment sets disagree. Let J be a single Γ -consistent and complete judgment set, and let $(J_1, \dots, J_p) = \mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ be a profile. We define the distance between J and \mathbf{J} to be $\text{Dist}(J, \mathbf{J}) = \sum_{i \in [p]} d(J, J_i)$. Then, we let the outcome $\text{Kemeny}_{\Phi, \Gamma}(\mathbf{J})$ of the Kemeny rule be the set of those $J^* \in \mathcal{J}(\Phi, \Gamma)$ for which there is no $J \in \mathcal{J}(\Phi, \Gamma)$ such that $\text{Dist}(J, \mathbf{J}) < \text{Dist}(J^*, \mathbf{J})$. (If Φ and Γ are clear from the context, we often write $\text{Kemeny}(\mathbf{J})$ to denote $\text{Kemeny}_{\Phi, \Gamma}(\mathbf{J})$.) Intuitively, the Kemeny rule selects those complete and Γ -consistent judgment sets that minimize the cumulative Hamming distance to the judgment sets in the profile. The Kemeny rule is irresolute, complete, Γ -consistent and anonymous.

2.2.2 Constraint-Based Judgment Aggregation

We continue with the framework of constraint-based judgment aggregation.

Let $\mathcal{I} = \{x_1, \dots, x_n\}$ be a finite set of *issues*, in the form of propositional variables. Intuitively, these issues are the topics about which the individuals want to combine their judgments. A truth assignment $\alpha : \mathcal{I} \rightarrow \mathbb{B}$ is called a *ballot*, and represents an opinion that individuals and the group can have. We will also denote ballots α by a binary vector $(b_1, \dots, b_n) \in \mathbb{B}^n$, where $b_i = \alpha(x_i)$ for each $i \in [n]$. Moreover, we say that $(p_1, \dots, p_n) \in \{0, 1, \star\}^n$ is a *partial ballot*, and that (p_1, \dots, p_n) *agrees with* a ballot (b_1, \dots, b_n) if $p_i = b_i$ whenever $p_i \neq \star$, for all $i \in [n]$. As in the case for formula-based judgment aggregation, we introduce an integrity constraint Γ , that can be used to restrict the set of feasible opinions (for both the individuals and the group). The integrity constraint Γ is a propositional formula on the variables x_1, \dots, x_n . We define the set $\mathcal{R}(\mathcal{I}, \Gamma)$ of *rational ballots* to be the ballots (for \mathcal{I}) that satisfy the integrity constraint Γ . Rational ballots in the constraint-based judgment aggregation framework correspond to complete and Γ -consistent judgment sets in the formula-based judgment aggregation framework. We say that finite sequences $\mathbf{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ of rational ballots are *profiles*, and where convenient we equate a profile $\mathbf{r} = (r_1, \dots, r_p)$ with the (multi)set $\{r_1, \dots, r_p\}$. Moreover, for $i \in [p]$, we let \mathbf{r}_{-i} denote the profile $(r_1, \dots, r_{i-1}, r_{i+1}, \dots, r_p)$.

A *judgment aggregation procedure* (or *rule*), for the set \mathcal{I} of issues and the integrity constraint Γ , is a function F that takes as input a profile $\mathbf{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$, and that produces a non-empty set of ballots. We call a judgment aggregation procedure F *resolute* if for any profile \mathbf{r} it returns a singleton, i.e., $|F(\mathbf{r})| = 1$; otherwise, we call F *irresolute*. We call a judgment aggregation procedure F *anonymous* if for every profile $\mathbf{r} = (r_1, \dots, r_p)$ and for every permutation $\pi : [p] \rightarrow [p]$ it holds that $F(\mathbf{r}) = F(\mathbf{r}')$, where $\mathbf{r}' = (r_{\pi(1)}, \dots, r_{\pi(p)})$. We call a judgment aggregation procedure F *rational* (or *consistent*), if r is rational for every $\mathbf{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ and every $r \in F(\mathbf{r})$.

As an example of a judgment aggregation procedure we consider the *strict majority rule* Majority, where $\text{Majority}(\mathbf{r}) = \{(b_1, \dots, b_n)\}$ and where each b_i agrees with the majority of the i -th bits in the ballots in \mathbf{r} (in case of a tie, we arbitrarily let $b_i = 0$). To see that Majority is not rational, consider the set $\mathcal{I} = \{x_1, x_2, x_3\}$ of issues, the integrity constraint $\Gamma = x_3 \leftrightarrow (x_1 \rightarrow x_2)$, and the profile $\mathbf{r} = (r_1, r_2, r_3)$, where $r_1 = (1, 1, 1)$, $r_2 = (1, 0, 0)$, and $r_3 = (0, 0, 1)$. The unique outcome $(1, 0, 1)$ in $\text{Majority}(\mathbf{r})$ is not rational.

The *Kemeny aggregation procedure* is defined for the constraint-based judgment aggregation framework as follows. Similarly to the case for formula-based judgment aggregation,

the Kemeny rule is based on the Hamming distance $d(r, r') = |\{i \in [n] : b_i \neq b'_i\}|$, between two rational ballots $r = (b_1, \dots, b_n)$ and $r' = (b'_1, \dots, b'_n)$ for the set \mathcal{I} of issues and the integrity constraint Γ . Let r be a single ballot, and let $(r_1, \dots, r_p) = \mathbf{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ be a profile. We define the distance between r and \mathbf{r} to be $\text{Dist}(r, \mathbf{r}) = \sum_{i \in [p]} d(r, r_i)$. Then, we let the outcome $\text{Kemeny}_{\mathcal{I}, \Gamma}(\mathbf{r})$ of the Kemeny rule be the set of those ballots $r^* \in \mathcal{R}(\mathcal{I}, \Gamma)$ for which there is no $r \in \mathcal{R}(\mathcal{I}, \Gamma)$ such that $\text{Dist}(r, \mathbf{r}) < \text{Dist}(r^*, \mathbf{r})$. (If \mathcal{I} and Γ are clear from the context, we often write $\text{Kemeny}(\mathbf{r})$ to denote $\text{Kemeny}_{\mathcal{I}, \Gamma}(\mathbf{r})$.) The Kemeny rule is irresolute, anonymous and rational.

2.2.3 Extended Constraint-Based Judgment Aggregation

Finally, we consider an extended variant of the constraint-based judgment aggregation framework. In the constraint-based judgment aggregation framework, we consider a set \mathcal{I} of issues and an integrity constraint in the form of a propositional formula Γ that satisfies the constraint that $\text{Var}(\Gamma) \subseteq \mathcal{I}$. However, in some situations it is more convenient to allow the integrity constraint Γ to contain additional variables. In the extended constraint-based framework, we relax the condition that $\text{Var}(\Gamma) \subseteq \mathcal{I}$, and we allow arbitrary propositional formulas as integrity constraints. This modification requires us to adapt the notion of rationality accordingly. A ballot $\alpha : \mathcal{I} \rightarrow \mathbb{B}$ is said to be *rational* if $\Gamma[\alpha]$ is satisfiable—that is, if there is some truth assignment $\beta : \text{Var}(\Gamma) \setminus \mathcal{I} \rightarrow \mathbb{B}$ such that $\alpha \cup \beta$ satisfies Γ .

2.2.4 Preferences over Opinions

Strategic behavior for judgment aggregation (such as the problems of manipulation, bribery and control) involves the incentive to obtain a “better” outcome. Therefore, in order to study strategic behavior, it is essential to define a notion of preference over opinions—that is, when is one opinion “better than” (or preferred over) another opinion.

In the worst case, the number of possible opinions that play a role is exponential in the number of issues—e.g., for m issues there could be up to 2^m possible opinions. As a result, it is unreasonable to expect agents to explicitly specify a preference relation over all (feasible) opinions. Instead it makes more sense to use a compact specification language to represent a preference relation over opinions. In this paper, we will use one such specification method that can be used to capture a wide range of preferences. Various preference relations over opinions have been studied in the literature [3, 6, 8].

We consider preferences based on a *weighted Hamming distance*. We define this weighted Hamming distance for the setting of formula-based judgment aggregation. Definitions for the setting of constraint-based judgment aggregation are entirely similar. Take an agenda Φ together with an integrity constraint Γ . An agent can specify their preference relation over complete and Γ -consistent judgment sets $J \in \mathcal{J}(\Phi, \Gamma)$ by providing a weight function $w : [\Phi] \rightarrow \mathbb{N}$ that produces a weight $w(\varphi)$ for each formula $\varphi \in [\Phi]$. Intuitively, for each $\varphi \in [\Phi]$, the weight $w(\varphi)$ indicates how important it is for the agent that the outcome agrees with their truthful opinion on the issue φ . (Alternatively, one could consider weight functions that produce rational or real weights.) Then, for two complete judgment sets J_1 and J_2 the weighted Hamming distance $d(J_1, J_2, w)$ is defined as follows:

$$d(J_1, J_2, w) = \sum \{w(\varphi) : \varphi \in [\Phi], \varphi \in (J_1 \setminus J_2) \cup (J_2 \setminus J_1)\}.$$

That is, for each formula $\varphi \in [\Phi]$ that J and J' disagree on, the weighted Hamming distance is increased by $w(\varphi)$.

Using this notion of weighted Hamming distance, we can define a preference relation for an agent. Suppose that the agent’s truthful opinion is given by a complete and Γ -consistent judgment set J . Moreover, suppose that the agent’s view on the relative importance of

the separate issues is given by a weight function $w : [\Phi] \rightarrow \mathbb{N}$. Then the preference relation $\leq_{w,J}$ for this agent is defined as follows. For any two complete and Γ -consistent judgment sets J_1, J_2 , it holds:

$$J_1 \leq_{w,J} J_2 \quad \text{if and only if} \quad d(J, J_1, w) \leq d(J, J_2, w).$$

Correspondingly, a judgment set J_1 is (strictly) preferred over another judgment set J_2 if and only if the weighted Hamming distance from J_1 to J is (strictly) smaller than the weighted Hamming distance from J_2 to J .

A particular case of the weighted Hamming distance is the *unweighted Hamming distance*. That is, the case where $w(\varphi) = 1$ for all $\varphi \in [\Phi]$. Whenever the weight function w is the constant function that always returns 1, we drop the “ w ” from the notation—that is, the unweighted Hamming distance between two judgment sets J_1 and J_2 is denoted by $d(J_1, J_2)$.

Other preference relations In the literature, there have been various proposals for notions of preference over opinions. For example, the phenomenon of manipulation in judgment aggregation has been studied in the settings (1) where one judgment set is preferred over another if it agrees with a fixed optimal judgment set on at least one issue where the other judgment set disagrees [8], and (2) where one judgment set is preferred over a second judgment set if it agrees with a fixed optimal judgment set on at least one issue where the second judgment set disagrees, and for all issues it holds that if the second judgment set agrees with the optimal judgment set then the first judgment set also agrees with the optimal [8]. Other preference relations that have been investigated are top-respecting preferences and closeness-respecting preferences. The class of top-respecting preferences contains all preferences that prefer a single most preferred judgment set over all other judgment sets (and the preference between the other judgment sets is arbitrary) [3, 6]. The class of closeness-respecting preferences contains preferences that additionally satisfy the condition of closeness: if one judgment set agrees with the most preferred judgment on a superset of issues compared to another judgment set, then the one judgment is preferred over the other [3, 6].

3 Manipulation

The first form of strategic behavior in judgment aggregation that we consider is manipulation. This concerns cases where individuals aim to influence the outcome of the aggregation procedure in their favor by reporting an insincere judgment, that is, by reporting a judgment that differs from their beliefs.

For irresolute judgment aggregation procedures such as the Kemeny procedure, one can consider various requirements on the strategically reported insincere judgments. For instance, one could require that every outcome for the insincere judgment is preferred over every outcome for the sincere judgment. Alternatively, one could require that there is at least one outcome for the insincere judgment that is preferred over every outcome for the sincere judgment. Correspondingly, we consider the following decision problems. (We formalize these problems for the setting of formula-based judgment aggregation. For the setting of constraint-based judgment aggregation, these problems are defined entirely similarly.)

CAUTIOUS-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ such that for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$?

OPTIMISTIC-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ and **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ such that for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$?

PESSIMISTIC-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ such that for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ there is **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$?

SUPEROPTIMISTIC-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$, **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$?

SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ such that (1) for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1, w) \leq d(J_{\text{old}}^*, J_1, w)$, and such that (2) there exists **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$?

3.1 Complexity Results

In this section, we prove the following result.

Theorem 1. *The following problems are Σ_2^P -complete:*

- CAUTIOUS-MANIPULATION(Kemeny),
- OPTIMISTIC-MANIPULATION(Kemeny),
- PESSIMISTIC-MANIPULATION(Kemeny),
- SUPEROPTIMISTIC-MANIPULATION(Kemeny), and
- SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny).

Moreover, Σ_2^P -hardness holds already for the case where the manipulator's preferences are based on the unweighted Hamming distance.

This result follows from Propositions 2–5 and 7, and Corollaries 6 and 8, that we establish below.

Proposition 2. CAUTIOUS-MANIPULATION(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. Let $(\Phi, \Gamma, w, \mathbf{J})$ specify an instance of CAUTIOUS-MANIPULATION(Kemeny). The algorithm proceeds in several steps.

Firstly, (1) the algorithm determines the minimum distance $d_{\text{old}}^{\text{win}}$ from \mathbf{J} to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. That is, $d_{\text{old}}^{\text{win}}$ is the cumulative unweighted Hamming distance from the judgments in \mathbf{J} to the judgment sets $J^* \in \text{Kemeny}(\mathbf{J})$. This can be done in (deterministic) polynomial time using $O(\log n)$ queries to an NP oracle.

Then, (2) the algorithm determines the minimum distance $d_{\text{old}}^{\text{min}}$ (weighted by w) from J_1 to any judgment set $J^* \in \text{Kemeny}(\mathbf{J})$, that is, from J_1 to any complete and consistent judgment set J^* that has cumulative unweighted Hamming distance $d_{\text{old}}^{\text{win}}$ to the profile \mathbf{J} . This can also be done in (deterministic) polynomial time using an NP oracle.

Next, (3a) the algorithm guesses a complete judgment set J' together with a truth assignment $\alpha : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α satisfies both J' and Γ . This can be done in nondeterministic polynomial time. Moreover, (3b) the algorithm determines the minimum distance $d_{\text{new}}^{\text{win}}$ from (\mathbf{J}_{-1}, J') to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. Finally, (3c) the algorithm determines by using a single query to an NP oracle whether there exists some complete and consistent judgment set $J_{\text{new}}^* \in \mathcal{J}(\Phi, \Gamma)$ such that $d(J_{\text{new}}^*, (\mathbf{J}_{-1}, J')) = d_{\text{new}}^{\text{win}}$ and $d(J_{\text{new}}^*, J_1, w) \geq d_{\text{old}}^{\text{min}}$. If this is the case, the algorithm rejects; otherwise, the algorithm accepts.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic choices) if and only if there is some complete and consistent judgment set J' such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$. \square

Proposition 3. OPTIMISTIC-MANIPULATION(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. Let $(\Phi, \Gamma, w, \mathbf{J})$ specify an instance of OPTIMISTIC-MANIPULATION(Kemeny). The algorithm proceeds in several steps. For the first two steps, the algorithm proceeds exactly as the algorithm described in the proof of Proposition 2. That is, (1) the algorithm computes $d_{\text{old}}^{\text{win}}$ and (2) it computes $d_{\text{old}}^{\text{min}}$, both in deterministic polynomial time using an NP oracle.

For the third step, the algorithm proceeds in a similar fashion as the algorithm described in the proof of Proposition 2. That is, (3a) the algorithm guesses a complete judgment set J' together with a truth assignment $\alpha : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α satisfies both J' and Γ . Also, (3b) the algorithm determines the minimum unweighted Hamming distance $d_{\text{new}}^{\text{win}}$ from (\mathbf{J}_{-1}, J') to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. This can be done using $O(\log n)$ queries to an NP oracle. Then, (3c') the algorithm guesses some complete judgment set J_{new}^* together with a truth assignment $\alpha' : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α' satisfies both J_{new}^* and Γ . Moreover, the algorithm checks whether $d(J_{\text{new}}^*, (\mathbf{J}_{-1}, J')) = d_{\text{new}}^{\text{win}}$ —that is, $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ —and $d(J_{\text{new}}^*, J_1, w) < d_{\text{old}}^{\text{min}}$, and accepts if and only if this is the case.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic choices) if and only if there is some complete and consistent judgment set J' and some $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ such that for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$. \square

Proposition 4. PESSIMISTIC-MANIPULATION(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. Let $(\Phi, \Gamma, w, \mathbf{J})$ specify an instance of PESSIMISTIC-MANIPULATION(Kemeny). The algorithm proceeds in several steps. For the first step, the algorithm proceeds exactly as the algorithm described in the proof of Proposition 2. That is, (1) the algorithm computes $d_{\text{old}}^{\text{win}}$ in deterministic polynomial time using $O(\log n)$ queries to an NP oracle.

Then, (2) the algorithm determines the maximum distance $d_{\text{old}}^{\text{max}}$ (weighted by w) from J_1 to any judgment set $J^* \in \text{Kemeny}(\mathbf{J})$, that is, from J_1 to any complete and consistent judgment set J^* that has cumulative unweighted Hamming distance $d_{\text{old}}^{\text{win}}$ to the profile \mathbf{J} . This can also be done in (deterministic) polynomial time using an NP oracle.

Next, (3a) the algorithm guesses a complete judgment set J' together with a truth assignment $\alpha : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α satisfies both J' and Γ . This can be done in nondeterministic polynomial time. Moreover, (3b) the algorithm determines the minimum unweighted Hamming distance $d_{\text{new}}^{\text{win}}$ from (\mathbf{J}_{-1}, J') to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. Finally, (3c) the algorithm determines by using a single query to an NP oracle whether there exists some complete and consistent judgment set $J_{\text{new}}^* \in \mathcal{J}(\Phi, \Gamma)$ such that $d(J_{\text{new}}^*, (\mathbf{J}_{-1}, J')) = d_{\text{new}}^{\text{win}}$ and $d(J_{\text{new}}^*, J_1, w) \geq d_{\text{old}}^{\text{max}}$. If this is the case, the algorithm rejects; otherwise, the algorithm accepts.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic choices) if and only if there is some complete and consistent judgment set J' such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ there is some $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$. \square

Proposition 5. SUPEROPTIMISTIC-MANIPULATION(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. Let $(\Phi, \Gamma, w, \mathbf{J})$ specify an instance of SUPEROPTIMISTIC-MANIPULATION(Kemeny). The algorithm proceeds in several steps. For the first steps, the algorithm proceeds exactly as the algorithm described in the proof of Proposition 2. That is, (1) the algorithm computes $d_{\text{old}}^{\text{win}}$, in deterministic polynomial time using $O(\log n)$ queries to an NP oracle.

Then, (2') the algorithm guesses a complete judgment set J_{old}^* together with a truth assignment $\gamma : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether γ satisfies both J_{old}^* and Γ , and whether $d(J_{\text{old}}^*, \mathbf{J}) = d_{\text{old}}^{\text{win}}$.

For the third step, the algorithm proceeds in a similar fashion as the algorithm described in the proof of Proposition 2. That is, (3a) the algorithm guesses a complete judgment set J' together with a truth assignment $\alpha : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α satisfies both J' and Γ . Also, (3b) the algorithm determines the minimum distance $d_{\text{new}}^{\text{win}}$ from (\mathbf{J}_{-1}, J') to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. This can be done using $O(\log n)$ queries to an NP oracle. Then, (3c'') the algorithm guesses some complete judgment set J_{new}^* together with a truth assignment $\alpha' : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks whether α' satisfies both J_{new}^* and Γ . Moreover, the algorithm checks whether $d(J_{\text{new}}^*, (\mathbf{J}_{-1}, J')) = d_{\text{new}}^{\text{win}}$ —that is, $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ —and $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$, and accepts if and only if this is the case.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic choices) if and only if there is some complete and consistent judgment set J' , some $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and some $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_1, w) < d(J_{\text{old}}^*, J_1, w)$. \square

Corollary 6. SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny) is in Σ_2^P .

Proof (sketch). The algorithms described in the proofs of Proposition 2 and 5 can straightforwardly be modified and combined to form a nondeterministic polynomial-time algorithm with access to an NP oracle that solves SAFE-SUPEROPTIMISTIC-MANIPULATION-(Kemeny). \square

Proposition 7. CAUTIOUS-MANIPULATION(Kemeny) is Σ_2^P -hard.

Proof. We show Σ_2^P -hardness by giving a reduction from the satisfiability problem for quantified Boolean formulas of the form $\exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$. Let $\varphi = \exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$ be a quantified Boolean formula. Let $X = \{x_1, \dots, x_n\}$ and $Y = \{y_1, \dots, y_m\}$. Without loss of generality, we assume that n is a multiple of 3—that is, that $n = 3n'$ for some $n' \in \mathbb{N}$. We construct an agenda Φ , an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, and a profile \mathbf{J} as follows.

We consider fresh variables x'_1, \dots, x'_n and fresh variables y'_1, \dots, y'_m . Moreover, we consider fresh variables $z_1, \dots, z_{n/3}$, fresh variables t_1, \dots, t_m , fresh variables $w_{1,\ell}$ for $\ell \in [n+1]$, fresh variables $w_{2,\ell}$ for $i \in \{2, 3\}$ and $\ell \in [n]$, and fresh variables $u_{i,\ell}$ for $i \in [3]$ and $\ell \in [u]$, where $u = 10n + 10m + 10$. We then let the agenda Φ consist of the variables x_1, \dots, x_n and y_1, \dots, y_m and the fresh variables we introduced above, together with their negations. That is, we let $[\Phi] = \{x_i, x'_i : i \in [n]\} \cup \{z_i : i \in [n/3]\} \cup \{y_j, y'_j, t_j : j \in [m]\} \cup \{w_{1,\ell} : \ell \in [n+1]\} \cup \{w_{i,\ell} : i \in \{2, 3\}, \ell \in [n]\} \cup \{u_{i,\ell} : i \in [3], \ell \in [u]\}$.

We then define the integrity constraint Γ as follows. We let

$$\Gamma = \Gamma_0 \vee \bigvee_{i \in [3]} \bigwedge_{\ell \in [u]} u_{i,\ell},$$

and

$$\begin{aligned} \Gamma_0 = & \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \right) \vee \bigwedge_{i \in [n/3]} z_i \right) \wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \right) \rightarrow \bigwedge_{i \in [n/3]} \neg z_i \right) \\ & \wedge \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \oplus \bigwedge_{j \in [m]} t_j \right) \wedge \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \rightarrow \bigwedge_{j \in [m]} \neg t_j \right) \\ & \wedge \left(\bigwedge_{i \in [n/3]} z_i \rightarrow \left(\bigwedge_{i \in [n]} (\neg x_i \wedge \neg x'_i) \right) \right) \wedge \left(\left(\bigwedge_{j \in [m]} t_j \right) \rightarrow \bigwedge_{j \in [m]} (\neg y_j \wedge \neg y'_j) \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n/3]} z_i \right) \rightarrow \bigwedge_{j \in [m]} t_j \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n/3]} z_i \wedge \bigwedge_{j \in [m]} t_j \right) \rightarrow \left(\bigwedge_{\ell \in [n]} w_{2,\ell} \vee \bigwedge_{\ell \in [n]} w_{3,\ell} \right) \right) \\ & \wedge \left(\left(\neg \left(\bigwedge_{i \in [n/3]} z_i \right) \wedge \bigwedge_{j \in [m]} t_j \right) \rightarrow \bigwedge_{\ell \in [n+1]} w_{1,\ell} \right) \\ & \wedge \left(\left(\neg \left(\bigwedge_{i \in [n/3]} z_i \right) \wedge \neg \left(\bigwedge_{j \in [m]} t_j \right) \right) \rightarrow \neg \psi \wedge \left(\bigwedge_{\ell \in [n]} w_{2,\ell} \vee \bigwedge_{\ell \in [n]} w_{3,\ell} \right) \right). \end{aligned}$$

(Here \oplus denotes exclusive disjunction.)

As a result of the definition of Γ , each complete and consistent judgment set $J \in \mathcal{J}(\Phi, \Gamma)$ satisfies (at least) one of the following four conditions.

1. For some $i \in [3]$, the judgment set J includes each formula $u_{i,\ell}$ for $\ell \in [u]$.
2. The judgment set J includes exactly one of x_i and x'_i for each $i \in [n]$, it includes none of the formulas z_i , it includes exactly one of y_j and y'_j for each $j \in [m]$, it includes none of the formulas t_j , it includes either all formulas $w_{2,\ell}$ or all formulas $w_{3,\ell}$ for $\ell \in [n]$, and J does not satisfy ψ .
3. The judgment set J includes exactly one of x_i and x'_i for each $i \in [n]$, it includes none of the formulas z_i , it includes all formulas t_j , for each $j \in [m]$ it includes either none or both of y_j and y'_j , and it includes all formulas $w_{1,\ell}$ for $\ell \in [n+1]$.
4. The judgment set J includes all formulas z_i , for each $i \in [n]$ it includes either none or both of x_i and x'_i , it includes all formulas t_j , for each $j \in [m]$ it includes either none or both of y_j and y'_j , and it includes either all formulas $w_{2,\ell}$ or all formulas $w_{3,\ell}$ for $\ell \in [n]$.

We define $w : [\Phi] \rightarrow \mathbb{N}$ by letting $w(\varphi) = 1$ for each $\varphi \in [\Phi]$. In other words, we consider the unweighted Hamming distance.

Finally, we let $\mathbf{J} = (J_1, J_2, J_3)$, where J_1, J_2, J_3 are defined as described in Table 1. In this table, the indices i, j, ℓ range over all possible values, and for each $\varphi \in [\Phi]$ we write a 0 if $\varphi \notin J_i$ and a 1 if $\varphi \in J_i$.

\mathbf{J}	x_i	x'_i	z_i	y_j	y'_j	t_j	$w_{1,\ell}$	$w_{2,\ell}$	$w_{3,\ell}$	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_1	0	0	0	0	0	0	1	0	0	1	0	0
J_2	0	0	0	0	0	0	0	1	0	0	1	0
J_3	1	1	0	0	0	0	0	0	1	0	0	1

Table 1: The profile $\mathbf{J} = (J_1, J_2, J_3)$ that we use in the proof of Proposition 7.

In the remainder, we will argue that there is some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true if and only if there is some complete and consistent judgment set $J'_1 \in \mathcal{J}(\Phi, \Gamma)$ such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J'_1)$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1) < d(J_{\text{old}}^*, J_1)$.

Firstly, we observe that $\text{Kemeny}(\mathbf{J}) = \{J_{\text{old},1}^*, J_{\text{old},2}^*\}$, where $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ are defined as described in Table 2. Both $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ are complete and consistent, and have a cumulative Hamming distance of $7n + 3m + 1 + 3u$ to the profile \mathbf{J} . It is straightforward to verify that no complete and consistent judgment set has a smaller cumulative Hamming distance to the profile \mathbf{J} . (For instance, the judgment sets of the form J_α^* , as described below in Table 3, have a cumulative Hamming distance of $7n + 3m + 2 + 3u$ to the profile \mathbf{J} .)

$\text{Kemeny}(\mathbf{J})$	x_i	x'_i	z_i	y_j	y'_j	t_j	$w_{1,\ell}$	$w_{2,\ell}$	$w_{3,\ell}$	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
$J_{\text{old},1}^*$	0	0	1	0	0	1	0	1	0	0	0	0
$J_{\text{old},2}^*$	0	0	1	0	0	1	0	0	1	0	0	0

Table 2: The judgment sets $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ that are used in the proof of Proposition 7.

Next, we argue that the only way for agent 1 to enforce an outcome that is closer to J_1 than $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ are, is to enforce an outcome J^* that satisfies condition (3). It is impossible for agent 1 to force a complete and consistent judgment set that satisfies condition (1) to be a Kemeny outcome, because there are complete and consistent judgment

sets at smaller distance to the profile for any judgment J'_1 that agent 1 reports (e.g., the judgment sets $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$). For each of the complete and consistent judgment sets that satisfy condition (2), the Hamming distance to J_1 is at least as much as the Hamming distance from J_1 to $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$. Among the complete and consistent judgment sets that satisfy condition (4), the judgment sets $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ are of smallest Hamming distance to J_1 . Therefore, there is no insincere judgment J'_1 that agent 1 can report to obtain a Kemeny outcome satisfying condition (4) that is of smaller Hamming distance to J_1 . This means that if agent 1 were to enforce a Kemeny outcome J_{new}^* that is of smaller Hamming distance to J_1 than $J_{\text{old},1}^*$ and $J_{\text{old},2}^*$ are (by reporting an insincere judgment J'_1), then J_{new}^* must satisfy condition (3).

We continue with observing several further properties that each complete and consistent judgment set $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ must satisfy. We know that J_{new}^* includes none of the formulas y_j and y'_j , as including these would strictly increase the Hamming distance to the profile. Similarly, J_{new}^* includes none of the formulas $w_{2,\ell}$, $w_{3,\ell}$ and $u_{i,\ell}$.

Finally, we argue that there is some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true if and only if there is some complete and consistent judgment set $J'_1 \in \mathcal{J}(\Phi, \Gamma)$ such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1) < d(J_{\text{old}}^*, J_1)$.

(\Rightarrow) Suppose that there exists some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. Consider the complete and consistent judgment set J_α that is described in Table 3. We then get that $\text{Kemeny}(\mathbf{J}_{-1}, J_\alpha) = \{J_\alpha^*\}$, where J_α^* is also described in Table 3. The only possible complete and consistent judgment sets J^* that could have a smaller Hamming distance to the profile $(\mathbf{J}_{-1}, J_\alpha)$ would have to satisfy condition (2). That is, such judgment sets J^* would have to include exactly one of y_j and y'_j , for each $j \in [m]$, and would have to satisfy $\neg\psi$. Moreover, in order for such judgment sets J^* to have a smaller Hamming distance to the profile, it would have to agree with J_α^* on the formulas x_i and x'_i , for all $i \in [n]$. However, since $\psi[\alpha]$ is valid, we know that such judgment sets J^* are not consistent. Therefore, $\text{Kemeny}(\mathbf{J}_{-1}, J_\alpha) = \{J_\alpha^*\}$. Clearly, $d(J_1, J_\alpha^*) < d(J_1, J_{\text{old},i}^*)$ for both $i \in [2]$. In other words, for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J_\alpha)$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1) < d(J_{\text{old}}^*, J_1)$.

	x_i	x'_i	z_i	y_j	y'_j	t_j	$w_{1,\ell}$	$w_{2,\ell}$	$w_{3,\ell}$	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_α	$\alpha(x_i)$	$1 - \alpha(x_i)$	0	0	0	1	1	0	0	1	0	0
J_α^*	$\alpha(x_i)$	$1 - \alpha(x_i)$	0	0	0	1	1	0	0	0	0	0

Table 3: The judgment sets J_α and J_α^* that are used in proof of Proposition 7.

(\Leftarrow) Conversely, suppose that there is some complete and consistent judgment set $J'_1 \in \mathcal{J}(\Phi, \Gamma)$ such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}_{-1}, J'_1)$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_1) < d(J_{\text{old}}^*, J_1)$. As we argued above, each such complete and consistent judgment set J_{new}^* satisfies condition (3), and furthermore, it does not contain any of the formulas $y_j, y'_j, w_{2,\ell}, w_{3,\ell}$ and $u_{i,\ell}$. That is, such a judgment set J_{new}^* must be of the form J_α^* , for some $\alpha : X \rightarrow \mathbb{B}$, as described in Figure 3. Fix this truth assignment $\alpha : X \rightarrow \mathbb{B}$. It suffices to consider insincere judgment sets J'_1 that minimize the distance to the judgment set J_α^* —that is, it suffices to consider the situation where $J'_1 = J_\alpha^*$. (The following argument works regardless of the judgment of J'_1 on the formulas $u_{1,\ell}$.) In other words, we consider the profile $\mathbf{J}' = (\mathbf{J}_{-1}, J_\alpha^*)$.

We now use the fact that $J_\alpha^* \in \text{Kemeny}(\mathbf{J}')$ to argue that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. We proceed indirectly, and assume that there exists some truth assignment $\beta : Y \rightarrow \mathbb{B}$ such that $\psi[\alpha \cup \beta]$ is false. Then consider the complete

	x_i	x'_i	z_i	y_j	y'_j	t_j	$w_{1,\ell}$	$w_{2,\ell}$	$w_{3,\ell}$	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_β^*	$\alpha(x_i)$	$1 - \alpha(x_i)$	0	$\beta(y_j)$	$1 - \beta(y_j)$	0	0	1	0	0	0	0

Table 4: The judgment set J_β^* that is used in the proof of Proposition 7.

and consistent judgment set J_β^* that is described in Table 4. Because $\psi[\alpha \cup \beta]$ is false, we know that J_β^* is consistent—it satisfies condition (2). Moreover, $d(J_\beta^*, \mathbf{J}') < d(J_\alpha^*, \mathbf{J}')$. Namely, the judgment set J_α^* has Hamming distance $6n + 3m + 2 + 2u$ to the profile \mathbf{J}' , and the judgment set J_β^* has Hamming distance $6n + 3m + 1 + 2u$ to the profile \mathbf{J}' . This is a contradiction with the fact that $J_\alpha^* \in \text{Kemeny}(\mathbf{J}')$. Therefore, we can conclude that no truth assignment $\beta : Y \rightarrow \mathbb{B}$ exists such that $\psi[\alpha \cup \beta]$ is false. In other words, for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. \square

Corollary 8. OPTIMISTIC-MANIPULATION(Kemeny), PESSIMISTIC-MANIPULATION(Kemeny), SUPEROPTIMISTIC-MANIPULATION(Kemeny), and SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny) are Σ_2^P -hard.

Proof. Σ_2^P -hardness for OPTIMISTIC-MANIPULATION(Kemeny), PESSIMISTIC-MANIPULATION(Kemeny), SUPEROPTIMISTIC-MANIPULATION(Kemeny), and SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny) follows from the proof of Proposition 7. In this proof, all Kemeny outcomes for the original profile \mathbf{J} have the same Hamming distance to the judgment set J_1 . Similarly, all Kemeny outcomes for each profile $\mathbf{J}' = (\mathbf{J}_{-1}, J'_1)$, for each relevant insincere judgment set J'_1 , have the same distance to the judgment set J_1 . Therefore, the reduction can also be used to show Σ_2^P -hardness for the problems OPTIMISTIC-MANIPULATION(Kemeny), PESSIMISTIC-MANIPULATION(Kemeny), SUPEROPTIMISTIC-MANIPULATION(Kemeny), and SAFE-SUPEROPTIMISTIC-MANIPULATION(Kemeny). \square

3.2 3-Clauses and Trivial Integrity Constraints

The result of Theorem 1 can straightforwardly be extended to the setting where $\Gamma = \top$ and where all formulas $\varphi \in [\Phi]$ are clauses of size 3. Clearly, membership in Σ_2^P holds also for this restricted setting. In order to show Σ_2^P -hardness, the proof of Proposition 7 can be modified as follows, in two steps. First, we take many syntactic variants χ_1, \dots, χ_u of the integrity constraint Γ , and add these (and their negations) to the agenda. Moreover, all judgment sets in the profile \mathbf{J} include these formulas χ_1, \dots, χ_u . Secondly, we transform each formula χ_i into a 3CNF formula χ'_i (by using the standard Tseitin transformation), and replace χ_i in the agenda by the clauses of χ'_i (and their negations). Then, all judgment sets in the profile \mathbf{J} include all clauses of all formulas χ'_i .

3.3 Constraint-Based Judgment Aggregation

The result of Theorem 1 can also straightforwardly be extended to the setting of constraint-based judgment aggregation. The algorithms described in Propositions 2–5 and Corollary 6 can directly be used in the setting of constraint-based judgment aggregation as well (so membership in Σ_2^P carries over to this setting). Since the proofs of Proposition 7 and Corollary 8 use an agenda containing only propositional variables (and their negations), this proof can also directly be used to show Σ_2^P -hardness for the setting of constraint-based judgment aggregation.

3.4 Exact Manipulation

In the literature, the problem of manipulation in judgment aggregation has also been modelled using different decision problems (see, e.g., [3]). One of these decision problems asks whether the manipulator can report an insincere opinion to obtain an outcome that includes a given desired subset of the agenda. This problem is often called *exact manipulation*. We consider two variants of this exact manipulation problem, and we argue that the Σ_2^P -completeness result of Theorem 1 extends to these problems.

CAUTIOUS-EXACT-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a (possibly incomplete) judgment set $L \subseteq \Phi$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ such that for **all** $J^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ it holds that $L \subseteq J^*$?

BRAVE-EXACT-MANIPULATION(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a (possibly incomplete) judgment set $L \subseteq \Phi$, and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there a complete and consistent judgment set $J' \in \mathcal{J}(\Phi, \Gamma)$ such that for **some** $J^* \in \text{Kemeny}(\mathbf{J}_{-1}, J')$ it holds that $L \subseteq J^*$?

Proposition 9. *The problems CAUTIOUS-EXACT-MANIPULATION(Kemeny) and BRAVE-EXACT-MANIPULATION(Kemeny) are Σ_2^P -complete. Moreover, Σ_2^P -hardness holds even for the case where $|L| = 1$.*

Proof. To show membership in Σ_2^P , it suffices to observe that the algorithms used in the proofs of Propositions 2 and 3 can straightforwardly be extended to the case of CAUTIOUS-EXACT-MANIPULATION(Kemeny) and BRAVE-EXACT-MANIPULATION(Kemeny). For Σ_2^P -hardness, one can use the proofs of Proposition 7 and Corollary 8, with the modification that $L = \{w_{1,1}\}$. \square

3.5 Group Manipulation

The result of Theorem 1 also holds for the setting where a coalition of individuals works together by each reporting an insincere judgment with the aim of obtaining a group judgment that is preferable for each of the individuals in the coalition [4]. This group manipulation scenario is a generalization of the (individual) manipulation scenario that we study in this paper, since individual manipulation coincides with group manipulation for a coalition of size 1. The algorithms used to show membership in Σ_2^P can be straightforwardly modified to work also for group manipulation for the Kemeny procedure. Moreover, the Σ_2^P -hardness results directly carry over to the setting of group manipulation for the Kemeny procedure, since individual manipulation is a special case of group manipulation.

A more subtle form of group manipulation is that where each individual in the coalition is required to have no incentive to unilaterally leave the coalition (and report their truthful judgment) [4]. The Σ_2^P -hardness results from this paper carry over to this setting for the Kemeny procedure, since the scenario of individual manipulation is also a special case of the manipulation problem for fragile coalitions. Additionally, the Σ_2^P -membership proofs for the individual manipulation problem can be modified to work also for the group manipulation problem for fragile coalitions for the Kemeny procedure.

4 Bribery

Another form of strategic behavior in judgment aggregation is bribery. In this setting, an external agent wishes to influence the outcome of a judgment aggregation scenario by bribing a number of individuals. We model this scenario as follows. (We consider the formula-based judgment aggregation framework; for the constraint-based judgment aggregation framework, this bribing scenario can be defined entirely similarly.)

A number of individuals are performing judgment aggregation on an agenda Φ in the presence of an integrity constraint Γ . The briber has a desired (complete and Γ -consistent) judgment set J_0 , and a weight function $w : [\Phi] \rightarrow \mathbb{N}$ that indicates the relative importance of the different issues for the briber. Additionally, the briber has a budget that suffices to bribe at most k individuals. For all bribed individuals, the briber can specify an arbitrary (complete and Γ -consistent) judgment set. The question is to determine whether the briber can pick up to k individuals and specify judgment sets for these individuals so that the outcome of the judgment aggregation procedure is better (with respect to J_0 and w) than without bribing.

To argue that such bribery can not be done easily in all possible situations, one can establish computational intractability results, that give evidence that there are no efficient algorithms that an external agent can use (across the board) to obtain a strategy for bribery.

Similarly to the case for manipulation, we can consider various requirements on the outcomes after bribery (in relation to the outcomes before bribery). For instance, we could require that every outcome after bribery is preferred over every outcome before bribery. Correspondingly, we consider the following decision problems.

CAUTIOUS-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a judgment set $J_0 \in \mathcal{J}(\Phi, \Gamma)$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$?

OPTIMISTIC-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a judgment set $J_0 \in \mathcal{J}(\Phi, \Gamma)$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that there is **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ such that for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$?

PESSIMISTIC-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a judgment set $J_0 \in \mathcal{J}(\Phi, \Gamma)$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ there is **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$?

SUPEROPTIMISTIC-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a judgment set $J_0 \in \mathcal{J}(\Phi, \Gamma)$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that there is **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$?

SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a judgment set $J_0 \in \mathcal{J}(\Phi, \Gamma)$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that (1) for **all** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for **all** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) \leq d(J_{\text{old}}^*, J_0, w)$, and so that (2) there exists **some** $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and **some** $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ such that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$?

4.1 Complexity Results

In this section, we prove the following result.

Theorem 10. *The following problems are Σ_2^P -complete:*

- CAUTIOUS-BRIBERY(Kemeny),
- OPTIMISTIC-BRIBERY(Kemeny),
- PESSIMISTIC-BRIBERY(Kemeny),
- SUPEROPTIMISTIC-BRIBERY(Kemeny), and
- SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny).

This result follows from Propositions 11 and 13 and Corollaries 12 and 14, that we establish below.

Proposition 11. CAUTIOUS-BRIBERY(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. This algorithm is similar to the algorithm used in the proof of Proposition 2. Let $(\Phi, \Gamma, w, \mathbf{J}, J_0, k)$ specify an instance of CAUTIOUS-BRIBERY(Kemeny), where $\mathbf{J} = (J_1, \dots, J_p)$. The algorithm proceeds in several steps.

Firstly, (1) the algorithm determines the minimum distance $d_{\text{old}}^{\text{win}}$ from \mathbf{J} to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. That is, $d_{\text{old}}^{\text{win}}$ is the cumulative unweighted Hamming distance from the judgments in \mathbf{J} to the judgment sets $J^* \in \text{Kemeny}(\mathbf{J})$. This can be done in (deterministic) polynomial time using $O(\log n)$ queries to an NP oracle.

Then, (2) the algorithm determines the minimum distance $d_{\text{old}}^{\text{min}}$ (weighted by w) from J_0 to any judgment set $J^* \in \text{Kemeny}(\mathbf{J})$, that is, from J_0 to any complete and consistent judgment set J^* that has cumulative unweighted Hamming distance $d_{\text{old}}^{\text{win}}$ to the profile \mathbf{J} . This can also be done in (deterministic) polynomial time using an NP oracle.

Next, (3a) the algorithm guesses k indices $1 \leq i_1 < \dots < i_k \leq p$ and guesses complete judgment sets J'_1, \dots, J'_k together with truth assignments $\alpha_1, \dots, \alpha_k : \text{Var}(\Phi, \Gamma) \rightarrow \mathbb{B}$, and it checks for each $j \in [k]$ whether α_j satisfies both J'_j and Γ . This can be done in nondeterministic polynomial time. The profile \mathbf{J}' is obtained from \mathbf{J} by replacing the judgment set J_{i_j} by J'_j , for each $j \in [k]$. Moreover, (3b) the algorithm determines the minimum distance $d_{\text{new}}^{\text{win}}$ from (\mathbf{J}') to any complete and consistent judgment set $J^* \in \mathcal{J}(\Phi, \Gamma)$. Finally, (3c) the algorithm determines by using a single query to an NP oracle whether there exists some complete and consistent judgment set $J_{\text{new}}^* \in \mathcal{J}(\Phi, \Gamma)$ such that $d(J_{\text{new}}^*, (\mathbf{J}')) = d_{\text{new}}^{\text{win}}$ and $d(J_{\text{new}}^*, J_0, w) \geq d_{\text{old}}^{\text{min}}$. If this is the case, the algorithm rejects; otherwise, the algorithm accepts.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic

choices) if and only if there is some way of replacing up to k judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , such that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$. \square

Corollary 12. OPTIMISTIC-BRIBERY(Kemeny), PESSIMISTIC-BRIBERY(Kemeny), SUPEROPTIMISTIC-BRIBERY(Kemeny), and SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny) are in Σ_2^P .

Proof (sketch). For each of these problems, a nondeterministic polynomial-time algorithm with access to an NP oracle can be constructed that solves the problem. These algorithms are analogous to the algorithms described in the proofs of Propositions 2–5 and 11, and Corollary 6. \square

Proposition 13. CAUTIOUS-BRIBERY(Kemeny) is Σ_2^P -hard.

Proof. We show Σ_2^P -hardness by giving a reduction from the satisfiability problem for quantified Boolean formulas of the form $\exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$. Let $\varphi = \exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$ be a quantified Boolean formula. Let $X = \{x_1, \dots, x_n\}$ and $Y = \{y_1, \dots, y_m\}$. We construct an agenda Φ , an integrity constraint Γ , a weight function $w : [\Phi] \rightarrow \mathbb{N}$, a profile \mathbf{J} as follows, a judgment set J_0 and an integer k as follows.

We consider the variables x_1, \dots, x_n and y_1, \dots, y_m and we introduce fresh variables x'_1, \dots, x'_n and y'_1, \dots, y'_m . Moreover, we introduce fresh variables z_i for $i \in [n]$, fresh variables t_j for $j \in [m]$, fresh variables $u_{i,\ell}$ for $i \in [3]$ and $\ell \in [u]$, where $u = 10n + 10m + 10$. Finally, we introduce a fresh variable a , and fresh variables b_j for $j \in [2m + 4]$. We then let $[\Phi] = \{x_i, x'_i, z_i : i \in [n]\} \cup \{y_j, y'_j, t_j : j \in [m]\} \cup \{u_{i,j} : i \in [3], j \in [u]\} \cup \{a, b_1, \dots, b_{2m+4}\}$.

We define the integrity constraint Γ as follows. We let

$$\Gamma = \Gamma_0 \vee \bigvee_{i \in [3]} \bigwedge_{\ell \in [u]} u_{i,\ell},$$

and

$$\begin{aligned} \Gamma_0 = & \left(a \vee \bigwedge_{j \in [2m+4]} b_j \right) \wedge \left(a \rightarrow \bigwedge_{j \in [2m+4]} \neg b_j \right) \wedge \left(\left(\bigwedge_{j \in [2m+4]} b_j \right) \rightarrow \neg a \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \right) \vee \bigwedge_{i \in [n]} z_i \right) \wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \right) \rightarrow \bigwedge_{i \in [n]} \neg z_i \right) \\ & \wedge \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \oplus \bigwedge_{j \in [m]} t_j \right) \wedge \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \rightarrow \bigwedge_{j \in [m]} \neg t_j \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n]} z_i \right) \rightarrow \left(\bigwedge_{i \in [n]} (\neg x_i \wedge \neg x'_i) \right) \right) \wedge \left(\left(\bigwedge_{j \in [m]} t_j \right) \rightarrow \bigwedge_{j \in [m]} (\neg y_j \wedge \neg y'_j) \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n]} z_i \right) \rightarrow \left(a \wedge \bigwedge_{j \in [m]} t_j \right) \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \wedge \bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \rightarrow (\neg \psi \wedge a) \right) \end{aligned}$$

$$\wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \wedge \bigwedge_{j \in [m]} t_j \right) \rightarrow \bigwedge_{j \in [2m+4]} b_j \right).$$

(Here \oplus denotes exclusive disjunction.)

As a result of the definition of Γ , each complete and consistent judgment set $J \in \mathcal{J}(\Phi, \Gamma)$ satisfies (at least) one of the following four conditions.

1. For some $i \in [3]$, the judgment set J includes each formula $u_{i,\ell}$ for $\ell \in [u]$.
2. The judgment set J includes exactly one of x_i and x'_i for each $i \in [n]$, it includes none of the formulas z_i , it includes exactly one of y_j and y'_j for each $j \in [m]$, it includes none of the formulas t_j , it includes a , it includes none of the formulas b_j , and J does not satisfy ψ .
3. The judgment set J includes exactly one of x_i and x'_i for each $i \in [n]$, it includes none of the formulas z_i , it includes all formulas t_j , for each $j \in [m]$ it includes either none or both of y_j and y'_j , it does not include a , and it includes all formulas b_j for $j \in [2m+4]$.
4. The judgment set J includes all formulas z_i , for each $i \in [n]$ it includes either none or both of x_i and x'_i , it includes all formulas t_j , for each $j \in [m]$ it includes either none or both of y_j and y'_j , it includes a , and it includes none of the formulas b_j .

We let $\mathbf{J} = (J_1, J_2, J_3)$, where J_1, J_2, J_3 are defined as described in Table 5. In this table, the indices i, j, ℓ range over all possible values, and for each $\varphi \in [\Phi]$ we write a 0 if $\varphi \notin J_i$ and a 1 if $\varphi \in J_i$.

\mathbf{J}	x_i	x'_i	z_i	y_j	y'_j	t_j	a	b_j	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_1	0	0	0	0	0	0	0	0	1	0	0
J_2	0	0	0	0	0	0	0	0	0	1	0
J_3	0	0	0	0	0	0	0	0	0	0	1

Table 5: The profile $\mathbf{J} = (J_1, J_2, J_3)$ that we use in the proof of Proposition 13.

Moreover, we let J_0 be as described in Table 6. In this table, the indices i, j, ℓ range over all possible values, and for each $\varphi \in [\Phi]$ we write a 0 if $\varphi \notin J_i$ and a 1 if $\varphi \in J_i$. In addition, we define the weight function w as follows. For each $j \in [2m+4]$, we let $w(b_j) = 1$, and for all other formulas $\varphi \in [\Phi]$, we let $w(\varphi) = 0$. In other words, the briber wants the formulas b_j to be satisfied, and does not care about any other formula.

	x_i	x'_i	z_i	y_j	y'_j	t_j	a	b_j	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_0	0	0	0	0	0	0	0	1	1	0	0

Table 6: The judgment set J_0 that we use in the proof of Proposition 13.

We let $k = 1$. That is, the briber can bribe exactly one individual.

In the remainder, we will argue that there is some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true if and only if it is possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$.

Firstly, we observe that $\text{Kemeny}(\mathbf{J}) = \{J_{\text{old}}^*\}$, where J_{old}^* is defined as described in Table 7. The judgment set J_{old}^* is complete and consistent, and has a cumulative Hamming distance of $6n + 3 + 3u$ to the profile \mathbf{J} . It is straightforward to verify that no complete and consistent judgment set has a smaller cumulative Hamming distance to the profile \mathbf{J} .

$\text{Kemeny}(\mathbf{J})$	x_i	x'_i	z_i	y_j	y'_j	t_j	a	b_j	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_{old}^*	0	0	1	0	0	1	1	0	0	0	0

Table 7: The judgment set J_{old}^* that is used in the proof of Proposition 13.

We observe that the only way for the briber to enforce an outcome that satisfies the formulas b_j , is to enforce an outcome J_{new}^* that satisfies condition (3). Additionally, we know that any such enforced outcome J_{new}^* includes none of the formulas y_j and y'_j , as a majority of judgment sets in the (modified) profile includes $\neg y_j$ and $\neg y'_j$. Similarly, any such enforced outcome J_{new}^* includes none of the formulas $u_{i,\ell}$.

We argue that there is some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true if and only if it is possible to change a single judgment set in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$.

(\Rightarrow) Suppose that there exists some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. Consider the complete and consistent judgment set J_α^* that is described in Table 8. Moreover, let \mathbf{J}' be the profile obtained from \mathbf{J} by replacing any single judgment set in \mathbf{J} by J_α^* (since the judgment sets in \mathbf{J} are symmetric, the argument does not depend on which set is replaced). We then get that $\text{Kemeny}(\mathbf{J}') = \{J_\alpha^*\}$. The only possible complete and consistent judgment sets J^* that could have a smaller Hamming distance to the profile (\mathbf{J}') would have to satisfy condition (2). That is, such judgment sets J^* would have to include exactly one of y_j and y'_j , for each $j \in [m]$, and would have to satisfy $\neg\psi$. Moreover, in order for such judgment sets J^* to have a smaller Hamming distance to the profile, it would have to agree with J_α^* on the formulas x_i and x'_i , for all $i \in [n]$. In particular, the cumulative unweighted Hamming distance from \mathbf{J}' to J_α^* is $2n + 6m + 8 + 2u$, and the cumulative unweighted Hamming distance from \mathbf{J}' to such a judgment set J^* would be $2n + 6m + 7 + 2u$. However, since $\psi[\alpha]$ is valid, we know that such judgment sets J^* are not consistent. Therefore, $\text{Kemeny}(\mathbf{J}') = \{J_\alpha^*\}$. Clearly, $d(J_0, J_\alpha^*, w) < d(J_0, J_{\text{old}}^*, w)$. In other words, for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$.

	x_i	x'_i	z_i	y_j	y'_j	t_j	a	b_j	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_α^*	$\alpha(x_i)$	$1 - \alpha(x_i)$	0	0	0	1	0	1	0	0	0

Table 8: The judgment set J_α^* that is used in proof of Proposition 13.

(\Leftarrow) Conversely, suppose that it is possible to change a single judgment set in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for all $J_{\text{new}}^* \in \text{Kemeny}(\mathbf{J}')$ and for all $J_{\text{old}}^* \in \text{Kemeny}(\mathbf{J})$ it holds that $d(J_{\text{new}}^*, J_0, w) < d(J_{\text{old}}^*, J_0, w)$. As we argued above, each such complete and consistent judgment set J_{new}^* satisfies condition (3), and furthermore, it does not contain any of the formulas y_j, y'_j and $u_{i,\ell}$. That is, such a judgment set J_{new}^* must be of the form J_α^* , for some $\alpha : X \rightarrow \mathbb{B}$, as described in Figure 8. Fix this truth assignment $\alpha : X \rightarrow \mathbb{B}$. It suffices to consider profiles \mathbf{J}' that are obtained from the profile \mathbf{J} by replacing one of the judgment

sets by J_α^* , as replacing a judgment set in \mathbf{J} with a judgment set that differs from J_α^* can only increase the cumulative unweighted Hamming distance to J_α^* .

	x_i	x'_i	z_i	y_j	y'_j	t_j	a	b_j	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_β^*	$\alpha(x_i)$	$1 - \alpha(x_i)$	0	$\beta(y_j)$	$1 - \beta(y_j)$	0	1	0	0	0	0

Table 9: The judgment set J_β^* that is used in the proof of Proposition 13.

We now use the fact that $J_\alpha^* \in \text{Kemeny}(\mathbf{J}')$ to argue that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. We proceed indirectly, and assume that there exists some truth assignment $\beta : Y \rightarrow \mathbb{B}$ such that $\psi[\alpha \cup \beta]$ is false. Then consider the complete and consistent judgment set J_β^* that is described in Table 9. Because $\psi[\alpha \cup \beta]$ is false, we know that J_β^* is consistent—it satisfies condition (2). Moreover, $d(J_\beta^*, \mathbf{J}') < d(J_\alpha^*, \mathbf{J}')$. Namely, the judgment set J_α^* has Hamming distance $2n + 6m + 8 + 2u$ to the profile \mathbf{J}' , and the judgment set J_β^* has Hamming distance $2n + 6m + 7 + 2u$ to the profile \mathbf{J}' . This is a contradiction with the fact that $J_\alpha^* \in \text{Kemeny}(\mathbf{J}')$. Therefore, we can conclude that no truth assignment $\beta : Y \rightarrow \mathbb{B}$ exists such that $\psi[\alpha \cup \beta]$ is false. In other words, for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. \square

Corollary 14. OPTIMISTIC-BRIBERY(Kemeny), PESSIMISTIC-BRIBERY(Kemeny), SUPEROPTIMISTIC-BRIBERY(Kemeny), and SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny) are Σ_2^P -hard.

Proof. Σ_2^P -hardness for OPTIMISTIC-BRIBERY(Kemeny), PESSIMISTIC-BRIBERY(Kemeny), SUPEROPTIMISTIC-BRIBERY(Kemeny), and SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny) follows from the proof of Proposition 13. In this proof, all Kemeny outcomes for the original profile \mathbf{J} have the same weighted Hamming distance to the judgment set J_0 . Similarly, all Kemeny outcomes for the relevant modified profiles \mathbf{J}' , have the same weighted Hamming distance to the judgment set J_0 . Therefore, the reduction can also be used to show Σ_2^P -hardness for the problems OPTIMISTIC-BRIBERY(Kemeny), PESSIMISTIC-BRIBERY(Kemeny), SUPEROPTIMISTIC-BRIBERY(Kemeny), and SAFE-SUPEROPTIMISTIC-BRIBERY(Kemeny). \square

4.2 Restricted Settings and Constraint-Based Judgment Aggregation

Similarly to the case of Theorem 1, the result of Theorem 10 can straightforwardly be extended to the setting where $\Gamma = \top$ and where all formulas $\varphi \in [\Phi]$ are clauses of size 3, by arguments that are entirely similar to the ones described in Section 3.2.

The result of Theorem 10 can also straightforwardly be extended to the setting of constraint-based judgment aggregation. The algorithms used in the proofs of Proposition 11 and Corollary 12 can directly be used in the setting of constraint-based judgment aggregation as well (so membership in Σ_2^P carries over to this setting). Moreover, since the proofs of Proposition 13 and Corollary 14 use an agenda containing only propositional variables (and their negations), this proof can also directly be used to show Σ_2^P -hardness for the setting of constraint-based judgment aggregation.

4.3 Exact Bribery

In the literature, the problem of bribery in judgment aggregation has also been modelled using different decision problems (see, e.g., [3]). One of these decision problems asks whether

the briber has a way of bribing a number of individuals (within their budget) to obtain an outcome that includes a given desired subset of the agenda. This problem is often called *exact bribery*. We consider two variants of this exact bribery problem, and we argue that the Σ_2^P -completeness result of Theorem 10 extends to these problems.

CAUTIOUS-EXACT-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a (possibly incomplete) judgment set $L \subseteq \Phi$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for **all** $J^* \in \text{Kemeny}(\mathbf{J}')$ it holds that $L \subseteq J^*$?

BRAVE-EXACT-BRIBERY(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$, a (possibly incomplete) judgment set $L \subseteq \Phi$, and an integer $k \in \mathbb{N}$.

Question: Is it possible to change up to k individual judgment sets in \mathbf{J} , resulting in a new profile \mathbf{J}' , so that for **some** $J^* \in \text{Kemeny}(\mathbf{J}')$ it holds that $L \subseteq J^*$?

Proposition 15. *The problems CAUTIOUS-EXACT-BRIBERY(Kemeny) and BRAVE-EXACT-BRIBERY(Kemeny) are Σ_2^P -complete. Moreover, Σ_2^P -hardness holds even for the case where $|L| = 1$.*

Proof. To show membership in Σ_2^P , it suffices to observe that the algorithms used in the proofs of Proposition 11 and Corollary 12 can straightforwardly be extended to the case of CAUTIOUS-EXACT-BRIBERY(Kemeny) and BRAVE-EXACT-BRIBERY(Kemeny). For Σ_2^P -hardness, one can use the proofs of Proposition 13 and Corollary 14, with the modification that $L = \{b_1\}$. \square

5 Control by Adding or Deleting Issues

A third form of strategic behavior in judgment aggregation is control. In this setting, an external agent wishes to influence the outcome of by influencing the conditions of a judgment aggregation scenario. Here, we consider control by (1) adding or (2) deleting issues. We model these scenarios as follows. (Again, we consider the formula-based judgment aggregation framework; for the constraint-based judgment aggregation framework, this control scenario can be defined entirely similarly.)

We begin with the scenario of (1) control by adding issues. A number of individuals each have an opinion for an agenda Φ in the presence of an integrity constraint Γ . That is, we are considering a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$. However, they are performing judgment aggregation only on a selection of issues, specified by an agenda $\Phi' \subseteq \Phi$. (For any $\Psi \subseteq \Phi$, we let the profile $\mathbf{J}|_\Psi$ consist of the judgment sets $J|_\Psi$ for each $J \in \mathbf{J}$, where $J|_\Psi = J \cap \Psi$ —that is, $\mathbf{J}|_\Psi = \{J \cap \Psi : J \in \mathbf{J}\}$. Intuitively, $\mathbf{J}|_\Psi$ is the restriction of \mathbf{J} to the formulas in Ψ .) The external agent wishes to ensure that the outcome of the judgment aggregation procedure includes a set $L \subseteq \Phi$ of conclusions, and they want to do so by enlarging the set of issues that the individuals perform judgment aggregation on. Formally, the external agent wants to select an agenda Φ'' with $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that $L \subseteq J^*$ for $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$. (Obviously, if the external agent wishes to succeed, they need to choose some Φ'' with $L \subseteq \Phi''$.) The question is whether the external agent can succeed in this. Since the Kemeny judgment aggregation procedure is irresolute, we can formulate two decision problems.

CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , an agenda $\Phi' \subseteq \Phi$, a set $L \subseteq \Phi$ and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ for Φ .

Question: Is there an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for **all** $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$?

BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , an agenda $\Phi' \subseteq \Phi$, a set $L \subseteq \Phi$ and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ for Φ .

Question: Is there an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for **some** $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$?

We continue with the scenario of (2) control by deleting issues. In this scenario, the individuals are performing judgment aggregation on an agenda Φ in the presence of an integrity constraint Γ . The external agent wishes to ensure that the outcome of the judgment aggregation procedure includes a set $L \subseteq \Phi$ of conclusions, and they want to do so by restricting the set of issues that the individuals perform judgment aggregation on. Moreover, several issues are outside the control of the agent—these issues cannot be excluded from the judgment aggregation scenario. This could be the case, for instance, if (1) the external agent has influence only on issues regarding one theme, or if (2) various issues are politically sensitive and excluding them would lead to controversy. This is modelled by a set $\Phi'' \subseteq \Phi$ of issues that the external agent cannot exclude. Formally, the external agent wants to select an agenda Φ'' with $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that $L \subseteq J^*$ for $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$. (Again, obviously, if the external agent wishes to succeed, they need to choose some Φ'' with $L \subseteq \Phi''$.) The question is whether the external agent can succeed in this. Again, since the Kemeny judgment aggregation procedure is irresolute, we can formulate two decision problems.

CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , an agenda $\Phi' \subseteq \Phi$, a set $L \subseteq \Phi$ and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for **all** $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$?

BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny)

Instance: An agenda Φ with an integrity constraint Γ , an agenda $\Phi' \subseteq \Phi$, a set $L \subseteq \Phi$ and a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$.

Question: Is there an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for **some** $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$?

Notice that the formulations of the decision problems CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny) and CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) are identical. However, the two different control scenarios that underlie these problems yield different restrictions of the problems that are interesting to investigate. For example, on the one hand, for the setting of control by deleting issues, it might often be the case that $\Phi' = \emptyset$ —that is, the external agent has full control on what issues to delete. On the other hand, for the setting of control by adding issues, it is reasonable to assume that $\Phi' \neq \emptyset$ —that is, the existing judgment aggregation scenario is not a trivial, vacuous one.

Unlike for the scenarios of manipulation and bribery, in this scenario of control we only formulated exact variants of the problem, where the objective is to obtain outcomes that include a given set L of conclusions, rather than obtaining outcomes that are preferable

according to a preference relation based on a weighted Hamming distance. This is due to the fact that in this control scenario, the agenda is not fixed. As a result, it is unclear what the agenda should be on which the weighted Hamming distance preferences are based.

5.1 Complexity Results

In this section, we prove the following result.

Theorem 16. *The following problems are Σ_2^P -complete:*

- CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny),
- BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny),
- CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny), and
- BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny).

Moreover, Σ_2^P -hardness holds even for the case where $\Phi' = \emptyset$.

This result follows from Propositions 17 and 19 and Corollaries 18 and 20–22, that we establish below.

Proposition 17. CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny) is in Σ_2^P .

Proof. We describe a nondeterministic polynomial-time algorithm with access to an NP oracle that solves the problem. Let $(\Phi, \Gamma, \Phi', L, \mathbf{J})$ specify an instance of CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny). The algorithm guesses an agenda Φ'' such that $\Phi' \subseteq \Phi'' \subseteq \Phi$. Then, the algorithm computes the minimum unweighted Hamming distance d^{win} from any judgment set that is Γ -consistent and complete for Φ'' to the profile $\mathbf{J}|_{\Phi''}$. This can be done in polynomial time using an NP oracle. Finally, the algorithm uses one more query to the NP oracle to decide if there exists a judgment set J^* that is Γ -consistent and complete for Φ'' that has Hamming distance d^{win} to the profile $\mathbf{J}|_{\Phi''}$ and that satisfies that $L \not\subseteq J^*$. The algorithm accepts if and only if no such judgment set J^* exists.

It is straightforward to verify that the algorithm runs in nondeterministic polynomial time. Moreover, the algorithm accepts the input (for some sequence of nondeterministic choices) if and only if there exists an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for all $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$. \square

Corollary 18. BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny) is in Σ_2^P .

Proof. The algorithm described in the proof of Proposition 17 can straightforwardly be modified to form a nondeterministic polynomial-time algorithm with access to an NP oracle that solves BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny). \square

Proposition 19. CAUTIOUS-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny) is Σ_2^P -hard.

Proof. We show Σ_2^P -hardness by giving a reduction from the satisfiability problem for quantified Boolean formulas of the form $\exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$. Let $\varphi = \exists x_1, \dots, x_n. \forall y_1, \dots, y_m. \psi$ be a quantified Boolean formula. Let $X = \{x_1, \dots, x_n\}$ and $Y = \{y_1, \dots, y_m\}$. Moreover, assume without loss of generality that ψ is in 3DNF. We construct an agenda Φ , an integrity constraint Γ , an agenda $\Phi' \subseteq \Phi$, a set $L \subseteq \Phi'$ a profile $\mathbf{J} \in \mathcal{J}(\Phi, \Gamma)^+$ as follows.

We introduce fresh variables $x_{i,1}, x_{i,2}, x_{i,3}, x'_{i,1}, x'_{i,2}, x'_{i,3}$ for each $i \in [n]$, we consider the variables y_j and we introduce fresh variables y'_j, t_j for each $j \in [m]$, we introduce fresh

variables a, b , and we introduce fresh variables $u_{i,\ell}$ for $i \in [3]$ and $\ell \in [u]$, where $u = 10n + 10m + 10$. We then let $[\Phi'] = \{y_j, y'_j, t_j : j \in [m]\} \cup \{a, b\} \cup \{u_{i,\ell} : i \in [3], \ell \in [u]\}$, and we let $[\Phi] = [\Phi'] \cup \{x_{i,j}, x'_{i,j} : i \in [n], j \in [3]\}$. Moreover, we let $L = \{b\}$.

We then define the integrity constraint Γ as follows. We let

$$\Gamma = \Gamma_0 \vee \bigvee_{i \in [3]} \bigwedge_{\ell \in [u]} u_{i,\ell},$$

and we let

$$\begin{aligned} \Gamma_0 = & (a \vee b) \wedge (\neg a \vee \neg b) \\ & \wedge \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \right) \vee \left(\bigwedge_{j \in [m]} t_j \right) \right) \\ & \wedge \left(\left(\bigwedge_{i \in [n]} (x_i \oplus x'_i) \right) \rightarrow \left(\left(\bigwedge_{j \in [m]} (y_j \oplus y'_j) \wedge a \wedge \neg \psi \right) \right. \right. \\ & \quad \left. \left. \vee \left(b \wedge \bigwedge_{j \in [m]} (t_j \wedge \neg y_j \wedge \neg y'_j) \right) \right) \right). \end{aligned}$$

(Here \oplus denotes exclusive disjunction.)

We let $\mathbf{J} = (J_1, J_2, J_3)$, where J_1, J_2, J_3 are defined as described in Table 10. In this table, the indices i, j, ℓ range over all possible values, and for each $\varphi \in [\Phi]$ we write a 0 if $\varphi \notin J_i$ and a 1 if $\varphi \in J_i$.

\mathbf{J}	$x_{i,j}$	$x'_{i,j}$	y_j	y'_j	t_j	a	b	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_1	1	1	0	0	0	0	0	1	0	0
J_2	1	1	0	0	0	0	0	0	1	0
J_3	1	1	0	0	0	1	0	0	0	1

Table 10: The profile $\mathbf{J} = (J_1, J_2, J_3)$ for the agenda Φ that we use in the proof of Proposition 19.

We observe that $\text{Kemeny}(\mathbf{J}|\Phi') = \{J_{\text{old},0}^*\} \cup \{J_{\text{old},\beta}^* : \beta : Y \rightarrow \mathbb{B}\}$, where the judgment sets $J_{\text{old},0}^*$ and $J_{\text{old},\beta}^*$ are defined as described in Table 11. The judgment sets $J_{\text{old},0}^*$ and $J_{\text{old},\beta}^*$ are complete (for the agenda Φ') and Γ -consistent, and have a cumulative Hamming distance of $3m + 2 + 3u$ to the profile $\mathbf{J}|\Phi'$. It is straightforward to verify that no complete and consistent judgment set has a smaller cumulative Hamming distance to the profile $\mathbf{J}|\Phi'$.

$\text{Kemeny}(\mathbf{J} \Phi')$	y_j	y'_j	t_j	a	b	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
$J_{\text{old},0}^*$	0	0	1	1	0	0	0	0
$J_{\text{old},\beta}^*$	$\beta(y_j)$	$1 - \beta(y_j)$	0	1	0	0	0	0

Table 11: The judgment set J_{old}^* that is used in the proof of Proposition 19.

We argue that there is some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true if and only if there is an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for all $J^* \in \text{Kemeny}(\mathbf{J}|\Phi'')$ it holds that $L \subseteq J^*$.

(\Rightarrow) Suppose that there exists some truth assignment $\alpha : X \rightarrow \mathbb{B}$ such that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. Consider the agenda Φ'' that is defined by letting $[\Phi''] = [\Phi'] \cup \{x_i : i \in [n], \alpha(x_i) = 1\} \cup \{x'_i : i \in [n], \alpha(x_i) = 0\}$. We argue that $\text{Kemeny}(\mathbf{J}|_{\Phi''}) = \{J_\alpha^*\}$, where J_α^* is defined as described in Table 12. The cumulative Hamming distance from J_α^* to $\mathbf{J}|_{\Phi''}$ is $3m + 3 + 3u$. Any complete (for the agenda Φ'') and Γ -consistent judgment set that would have less cumulative Hamming distance to $\mathbf{J}|_{\Phi''}$ would be of the form J_β^* , for some $\beta : Y \rightarrow \mathbb{B}$, as described in Table 12. Any such judgment set J_β^* would have a cumulative Hamming distance of $3m + 2 + 3u$ to the profile $\mathbf{J}|_{\Phi''}$. However, since $\psi[\alpha]$ is valid, we know that J_β^* is not Γ -consistent. Therefore, $\text{Kemeny}(\mathbf{J}|_{\Phi''}) = \{J_\alpha^*\}$. Moreover $L \subseteq J_\alpha^*$. Thus, we know that there exists an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for all $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$.

	$x_{i,j}$	$x'_{i,j}$	y_j	y'_j	t_j	a	b	$u_{1,\ell}$	$u_{2,\ell}$	$u_{3,\ell}$
J_α^*	1	1	0	0	1	0	1	0	0	0
J_β^*	1	1	$\beta(y_j)$	$1 - \beta(y_j)$	0	1	0	0	0	0

Table 12: The judgment sets J_α^*, J_β^* for the agenda Φ'' that is used in the proof of Proposition 19.

(\Leftarrow) Conversely, suppose that there exists an agenda $\Phi' \subseteq \Phi'' \subseteq \Phi$ such that for all $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ it holds that $L \subseteq J^*$. By construction of Γ , we know that Φ'' must contain exactly one of x_i and x'_i , for each $i \in [n]$. If this were not the case, one could take some set $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$ and construct the set $J' = J^* \setminus \{b, \neg a\} \cup \{a, \neg b\}$. This judgment set J' has a strictly smaller cumulative Hamming distance to the profile $\mathbf{J}|_{\Phi''}$, and is complete and Γ -consistent. This is a contradiction with the fact that $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$.

Now consider the truth assignment $\alpha : X \rightarrow \mathbb{B}$ that is defined by letting $\alpha(x_i) = 1$ if and only if $x_i \in \Phi''$. We claim that $\psi[\alpha]$ is valid, i.e., that for all truth assignments $\beta : Y \rightarrow \mathbb{B}$ it holds that $\psi[\alpha \cup \beta]$ is true. Suppose that there exists a truth assignment $\beta : Y \rightarrow \mathbb{B}$ such that $\psi[\alpha \cup \beta]$ is false. Moreover, consider the judgment set J_β^* that is defined as described in Table 12. This judgment set has a strictly smaller cumulative Hamming distance to the profile $\mathbf{J}|_{\Phi''}$ than any set $J^* \in \text{Kemeny}(\mathbf{J}|_{\Phi''})$. Moreover, since $\psi[\alpha \cup \beta]$ is false, we know that J_β^* is Γ -consistent. This is a contradiction, and thus we can conclude that there exists no truth assignment $\beta : Y \rightarrow \mathbb{B}$ such that $\psi[\alpha \cup \beta]$ is false. In other words, $\psi[\alpha]$ is valid. \square

Corollary 20. BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny) is Σ_2^P -hard.

Proof. Σ_2^P -hardness for the problem BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny), follows from the proof of Proposition 19. In this proof, either all Kemeny outcomes for the profile $\mathbf{J}|_{\Phi''}$ include the set L or no Kemeny outcomes for the profile $\mathbf{J}|_{\Phi''}$ include the set L . Therefore, the reduction can also be used to show Σ_2^P -hardness for BRAVE-EXACT-CONTROL-BY-ADDING-ISSUES(Kemeny). \square

Corollary 21. CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) and BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) are in Σ_2^P .

Proof. The algorithm described in the proof of Proposition 17 can straightforwardly be modified to form nondeterministic polynomial-time algorithms with access to an NP oracle that solve CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) and BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny). \square

Corollary 22. CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) and BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) are Σ_2^P -hard, even for the case where $\Phi' = \emptyset$.

Proof. Σ_2^P -hardness for CAUTIOUS-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) and BRAVE-EXACT-CONTROL-BY-DELETING-ISSUES(Kemeny) for the case where $\Phi' = \emptyset$ follows from the proof of Proposition 19. In the setting used in this hardness proof, the only way to achieve Kemeny outcomes J^* with $L \subseteq J^*$ is to choose agendas Φ'' of the form described in the proof. Therefore, the Σ_2^P -hardness proof works even for the case where $\Phi' = \emptyset$. \square

5.2 Constraint-Based Judgment Aggregation

The scenario of control by adding or deleting issues can also be formulated in the constraint-based judgment aggregation framework. In this case, it makes most sense to consider the extended variant of the constraint-based judgment aggregation framework where the integrity constraint Γ is allowed to contain propositional variables that are not contained in the set \mathcal{I} of issues. Because in the scenario of control by adding or deleting issues, the integrity constraint is fixed, whereas the set of issues is subject to control by the external agent, it is unclear how to adapt the integrity constraint after the set of issues has been changed. In the extended constraint-based judgment aggregation framework, there is no need to adapt the integrity constraint after the set of issues has been changed.

The result of Theorem 16 also holds in the extended variant of the constraint-based judgment aggregation framework. The membership results of Proposition 17 and Corollaries 18 and 21 also apply to this setting. Moreover, the proofs of the hardness results of Proposition 17 and Corollaries 20 and 22 involve agendas containing only propositional variables and their negations. Therefore, these hardness results also work for the setting of constraint-based judgment aggregation.

6 Conclusion

We investigated several decision problems that formalize the problems of manipulation, bribery and control (by adding or deleting issues) for the Kemeny judgment aggregation procedure. We showed that these problems are Σ_2^P -complete in their general formulation. The intractability results that we developed in this paper open up a wide range of natural questions for future research.

It would be interesting to study to what extent these intractability results for strategic behavior for the Kemeny rule hold up in a more refined computational complexity analysis. That is, do these intractability results also hold (1) when we consider restricted logic languages to specify the relation between issues and (2) when we use the more refined framework of parameterized complexity to capture natural restricted settings of the various problems. In order to compare different judgment aggregation procedures on the basis of the computational complexity of strategic behavior, it is necessary to also study these properties for further judgment aggregation procedures. Another natural direction for future research is to study variants of strategic behavior that are based on different notions of preference over judgment sets that have been considered in the literature. Finally, the types of strategic behavior that we considered in this paper are not the only relevant types that one needs to consider. An example of strategic behavior that we did not consider in this paper is control by adding or removing individuals (rather than adding or removing issues). It would be interesting to study such additional notions of strategic behavior in judgment aggregation from a computational complexity point of view as well.

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